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STEADY THREE-DIMENSIONAL TEMPERATURE FIELD IN COOLED TURBINE BLADES

V. I. Lokai and É. I. Gunchenko

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A method based on the use of Green functions is proposed for calculating the temperature field of cooled turbine blades. The method presumes the use of high-speed computers with large memories.

The creation of stoichiometric gas turbine engines and installations requires the solution of complex Scientific—technical problems. One of them is the reliable detailed calculation of the three-dimensional fields of temperatures and stresses in cooled turbine blades.

With an increase in the gas temperatures and intensification of the cooling the temperature gradients increase both over the height of the blades (especially in the basal zone) and through the cross section (especially in the zone of the edges and perforations). Under these conditions solutions based on the separation of the three-dimensional problem into one-dimensional and two-dimensional problems [1, 2] can lead to considerable errors.

An approximate solution of the three-dimensional problem of steady heat conduction in application to turbine blades with open cooling, reduced to the stage of practical use in contrast to [3], is presented in the present report. Such a problem comes down to integration in a simply connected region (the body of the blade) surrounded by a continuous medium (the gas and the coolant) with locally varying parameters: the temperature $T^*_{\text{sur.med}}$ (from T^*_g to T^*_{cool}) and the heat-transfer coefficients α (from α_g on the gas side to α_{cool} on the coolant side).

The solution described below is also valid for blades with a closed cooling system (a multiply connected region). The method presumes the use of a computer.

In the first approximation we solve the three-dimensional problem of heat conduction with $\lambda(T)$ = const, i.e., the system

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0, \tag{1}$$

$$\alpha (T^*_{\text{sur.med}} - T_s)\, dF = \lambda \frac{\partial T_s}{\partial n_s}\bigg|_{n_s=0} dF. \tag{2}$$

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Here T is the unknown temperature; x, y, and z are the coordinates of points of the body; Eq. (2) is the boundary condition on the side of the surrounding medium (the gas and coolant); T_s is the surface temperature of the wall; n_s is the normal to the surface of the body.

In accordance with the Green equation [4] the temperature of an arbitrary point k of the surface S of a blade which satisfies the system (1), (2) can be represented by the expression

$$T_{sk} = \frac{1}{4\pi} \iint\limits_{(\Sigma s)} \left[\frac{1}{r} \frac{\partial T_s}{\partial n_s} - T_s \frac{\partial \left(\frac{1}{r}\right)}{\partial n_s} \right] dS, \tag{3}$$

where the radius-vector $r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$ is the distance between a fixed point $M_0(x_0, y_0, z_0)$ and the current point $M(x, y, z)$.

The approximate calculation of the temperatures T_{sk} on the basis of (3) is carried out by the Fredholm method [5]. For this the entire surface of the blade is divided into n elementary areas and the integrals in (3) are replaced by finite sums. As a result, we obtain

$$4\pi T_{sk} = \sum_{i=1}^{n} \frac{\alpha_i}{\lambda} (T_{\text{sur.med}} - T_{si}) \iint\limits_{(S_i)} \frac{1}{r} dS - \sum_{i=1}^{n} T_{si} \iint\limits_{(S_i)} \frac{\partial \left(\frac{1}{r}\right)}{\partial n_s} dS. \tag{4}$$

The quantities α, $T^*_{\text{sur.med}}$, and T_s are taken outside the integral sign, since we use the assumption that they are constant within each elementary area of the partition.

We designate the coefficients to the unknown temperature T_s in (4) as Ψ and the coefficients to the stagnation temperatures $T^*_{\text{sur.med}}$ of the medium as $\Psi_{\text{sur.med}}$. Then in place of (4) we obtain a closed system of n linear algebraic equations for the determination of the temperatures at the surfaces of the n elementary areas of the partition:

$$\sum_{i=1}^{n} \Psi_{ki} T_{si} + 4\pi T_{s(i=k)} = \Psi_{\text{sur.med}\, k}, \tag{5}$$

where

$$\Psi_{ki} = \frac{\alpha_i}{\lambda} \iint\limits_{(S_i)} \frac{1}{r} dS + \iint\limits_{(S_i)} \frac{\partial \left(\frac{1}{r}\right)}{\partial n_s} dS; \tag{6}$$

$$\Psi_{\text{sur.med}\, k} = \frac{1}{\lambda} \sum_{i=1}^{n} \alpha_i T^*_{\text{sur.med}\, i} \iint\limits_{(S_i)} \frac{1}{r} dS, \quad k = 1, 2, 3, \ldots, n. \tag{7}$$

The temperatures at any point of the volume of the blade are found from the equation

$$T = \frac{1}{4\pi} \left[\Psi_{\text{sur. med}} - \sum_{i=1}^{n} \Psi_i T_{si}\right]. \tag{8}$$

The second approximation is satisfied with allowance for the dependence of the coefficient of thermal conductivity λ on the temperature T. For heat-resistant materials in the working temperature range one can take $\lambda = \lambda(T) = a + bT$ with sufficient accuracy. After substitution of variables and linearization of the boundary conditions in a way analogous to what was done in [2], in place of (1) and (2) we obtain

$$\frac{\partial^2 \Lambda}{\partial x^2} + \frac{\partial^2 \Lambda}{\partial y^2} + \frac{\partial^2 \Lambda}{\partial z^2} = 0, \tag{9}$$

$$\frac{\alpha}{\sqrt{\Lambda_1}} (2\sqrt{\Lambda_{\text{sur.med}} \Lambda_1} - \Lambda_1 - \Lambda) = \frac{\partial \Lambda}{\partial n_s}. \tag{10}$$

Here

$$\Lambda = \lambda^2 = (a + bT)^2; \quad T = \frac{\Lambda - \Lambda_1}{2b\sqrt{\Lambda_1}} + T_1;$$

$$\Lambda_1 = \lambda_1^2 = (a + bT_1)^2; \quad \Lambda_{\text{sur.med}} = \lambda^2_{\text{sur.med}} = (a + bT^*_{\text{sur.med}})^2;$$

T_1 is the blade temperature from the first approximation.

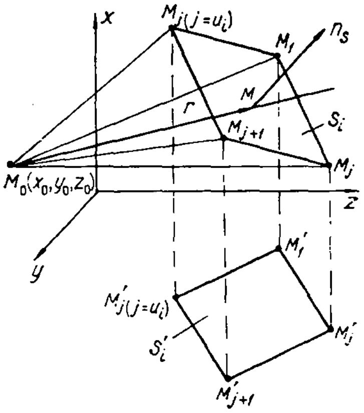

Fig. 1

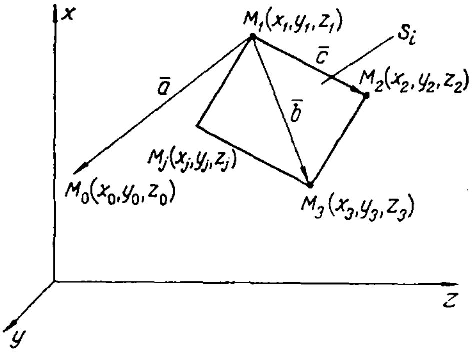

Fig. 2

Fig. 1. Explanatory diagram of calculation of surface integrals.

Fig. 2. Vector diagram for determination of direction of outer normal to the surface.

For the second approximation instead of (3) we obtain

$$\Lambda_{sk} = -\frac{1}{4\pi}\iint_{(\Sigma S)}\left[\frac{\alpha}{\sqrt{\Lambda_1}}\left(2\sqrt{\Lambda_{\text{sur. med}}}\,\Lambda_1 - \Lambda_1 - \Lambda_s\right)\frac{1}{r} - \Lambda_s\frac{\partial\left(\frac{1}{r}\right)}{\partial n_s}\right]dS\ . \tag{11}$$

The values of Λ_s at the surface of the blade are determined similarly to (4):

$$4\pi\Lambda_{sk} = \sum_{i=1}^{n}\frac{\alpha_i}{\lambda_{1i}}\left(2\lambda_{\text{sur.med}}\lambda_{1i} - \lambda_{1i}^2 - \Lambda_{si}\right)\iint_{(S_i)}\frac{1}{r}\,dS - \sum_{i=1}^{n}\Lambda_{si}\iint_{(S_i)}\frac{\partial\left(\frac{1}{r}\right)}{\partial n_s}\,dS. \tag{12}$$

After introducing the designations

$$\sigma_{ki} = \frac{\alpha_i}{\lambda_{1i}}\iint_{(S_i)}\frac{1}{r}\,dS + \iint_{(S_i)}\frac{\partial\left(\frac{1}{r}\right)}{\partial n_s}\,dS, \tag{13}$$

$$\sigma_{\text{sur.med}\,k} = \sum_{i=1}^{n}\alpha_i\left(2\lambda_{\text{sur.med}\,i} - \lambda_{1i}\right)\iint_{(S_i)}\frac{1}{r}\,dS \tag{14}$$

we obtain in place of (12) a system of linear algebraic equations for the determination of Λ_s at all n elementary areas of the partition

$$\sum_{i=1}^{n}\sigma_{ki}\Lambda_{si} + 4\pi\Lambda_{si(i=k)} = \sigma_{\text{sur.med}\,k},\quad k = 1,\ 2,\ \ldots,\ n. \tag{15}$$

The temperatures T at internal points of the volume are determined from the equation

$$T = \frac{\sqrt{\Lambda} - a}{b}\ \text{, where}\ \Lambda = -\frac{1}{4\pi}\left[\sum_{i=1}^{n}\sigma_i\Lambda_{si} - \sigma_{\text{sur. med}}\right] \tag{16}$$

A third and subsequent approximations can be carried out as necessary by analogy with the above.

The systems of equations (5) and (15) include integrals which are parts of the coefficients to the unknown quantities T_{si} and Λ_{si}

$$\iint\limits_{(S_i)} \frac{\partial\left(\frac{1}{r}\right)}{\partial n_s} dS = -\iint\limits_{(S_i)} \frac{1}{r^2} \frac{\partial r}{\partial n_s} dS = -\iint\limits_{(S_i)} \frac{1}{r^2} \cos(\widehat{r n_s})\, dS = -\varphi_{ki}. \tag{17}$$

The numerical value of such an integral is equal to the solid angle (φ_{ki}) whose apex is located at the fixed point $M_0(x_0, y_0, z_0)$ at the center of the k-th area of the partition of the blade surface S, while the solid angle itself has its base on the area S_i (Fig. 1). If the radius-vector from the point $M_0(x_0, y_0, z_0)$ reaches the area S_i from the internal region (through the material of the blade) then the quantity φ_{ki} is taken with a "plus" sign [$\cos\cdot(\widehat{rn_s}) > 0$], while if it reaches S_i from the external region then φ_{ki} is taken with a "minus" sign [$\cos(\widehat{rn_s}) < 0$].

The modulus of the solid angle φ_{ki} is calculated as a polyhedral angle by the usual methods of analytical geometry. The number of faces u_i of the polyhedral angle (see Fig. 1) is obviously equal to the number of straight lines bounding the area S_i. If in the preparation of the initial data the points bounding the area S_i are numbered counterclockwise (when looking at the surface from the outside), then in this case the sign of the solid angle can be determined from the sign of the scalar triple product of the vectors [6] $\bar{a}$, $\bar{b}$, and $\bar{c}$ (Fig. 2); i.e.,

$$\operatorname{sign} \varphi_{ki} = \operatorname{sign} [\bar{b}\ \bar{c}]\,\bar{a} = \operatorname{sign} \begin{vmatrix} x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_0 - x_1 & y_0 - y_1 & z_0 - z_1 \end{vmatrix}. \tag{18}$$

The double integral of 1/r over the surface of the area S_i is reduced to a double integral over the surface of its projection S_i' onto the coordinate plane zy (see Fig. 1):

$$\iint\limits_{(S_i)} \frac{1}{r} dS = \sqrt{1 + p^2 + q^2} \iint\limits_{(S_i')} \frac{1}{r} dS' = \sqrt{1 + p^2 + q^2} \iint\limits_{(S_i')} \frac{1}{r} dy dz. \tag{19}$$

Here $p = \partial x/\partial y$ and $q = \partial x/\partial z$ are the angular coefficients of the equation $x = py + qz + k$ of the plane to which the area S_i belongs.

Expressing the radius-vector r through the Cartesian coordinates and integrating (19) first over z and then over y, we finally obtain

$$\begin{aligned} \iint\limits_{(S_i)} \frac{1}{r} dS = \frac{\sqrt{G}}{\sqrt{A}} \sum_{j=1}^{u_i} \Bigg\{ & \left(V - \frac{M_j R_j}{T_j G}\right) \ln(\Pi_j t^2 + 2R_j t + H_j) - \\ & - V[1 + \ln(2AT_j)] + \left(\frac{R_j}{T_j(\sqrt{AT_j} - M_j)} - V\right) \ln t + \\ & + \frac{2|W|\sqrt{A}}{G} \operatorname{arctg} \frac{\Pi_j t + R_j}{\sqrt{\Delta_j}} \Bigg\} \Bigg|_{t_j}^{t_{j+1}}, \end{aligned} \tag{20}$$

where

$$t_j = y_j + \frac{L_j}{T_j} + \sqrt{\left(y_j + \frac{L_j}{T_j}\right)^2 + m_j}; \quad t_{j+1} = y_{j+1} + \frac{L_j}{T_j} + \sqrt{\left(y_{i+1} + \frac{L_j}{T_j}\right)^2 + m_j};$$

$$A = 1 + q^2; \ G = 1 + p^2 + q^2; \ W = py_0 + qz_0 + k - x_0, \ V = \frac{t^2 - m_j}{2t};$$

$$m_j = \frac{T_j \Phi_j - L_j^2}{T_j^2}; \ \Pi_j = T_j(\sqrt{AT_j} + M_j); \ H_j = m_j T_j(\sqrt{AT_j} - M_j);$$

$$R_j = N_j T_j - M_j L_j; \ \Delta = \Pi_j H_j - R_j^2 = T_j W^2 \geqslant 0; \ M_j = pq + a_j A;$$

$$N_j = Ac_j + q(k - x_0) - z_0; \ \Phi_j = (qc_j + k - x_0)^2 + (c_j - z_0)^2 + y_0^2;$$

$$T_j = 1 + a_j^2 + (qa_i + p)^2; \ L_j = a_j(c_j - z_0) + (a_j q + p)(qc_j + k - x_0) - y_0;$$

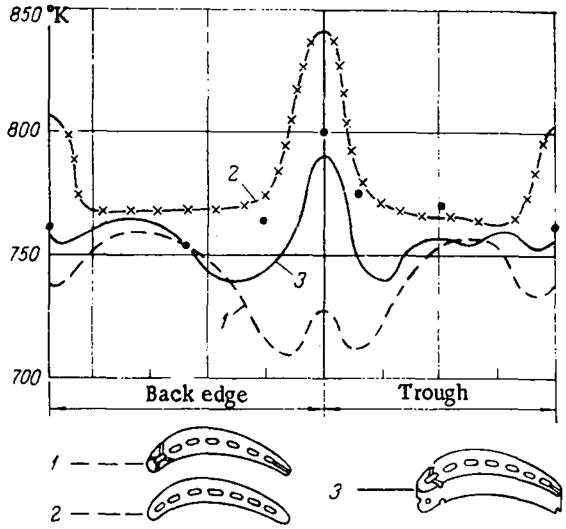

Fig. 3. Results of calculated—experimental test of the method developed.

a_j and c_j are from the equation for a straight line in the zy plane, i.e., $z = ay + c$,

$$a_j = \frac{z_{j+1} - z_j}{y_{j+1} - y_j}; \quad c_j = z_j - y_j \frac{z_{j+1} - z_j}{y_{j+1} - y_j}.$$

Upon closure of the integration contour t_1 is taken as t_{j+1} ($j = u_i$), where u_i is the number of straight lines bounding the area S_i'.

If the fixed point $M_0(x_0, y_0, z_0)$ belongs to the same plane as the area S_i, i.e., $W = 0$, then

$$\iint_{(S_i)} \frac{1}{r} dS = \frac{\sqrt{G}}{\sqrt{A}} \sum_{j=1}^{u_i} \Big\{ \Big(V - \frac{M_j R_j}{T_j G} \Big) \ln (\Pi_j t^2 + 2R_j t + H_j) - V[1 + \ln (2AT_j)] + \Big(\frac{R_j}{T_j (\sqrt{AT_j} - M_j)} - V \Big) \ln t \Big\} \Big|_{t_j}^{t_{j+1}}, \tag{21}$$

$$\iint_{(S_i)} \frac{\partial \left(\frac{1}{r}\right)}{\partial n_s} dS = -\varphi_{ki} = 0. \tag{22}$$

In the diagonal elements ($i = k$) of the systems (5) and (15) the quantity $\iint_{(S_i)} \frac{1}{r} dS$ is calculated from Eq. (21), while $\varphi_{ki(i=k)} = -2\pi$, since the point $M_0(x_0, y_0, z_0)$ lies inside the region of integration.

We emphasize that the integrals for Eqs. (20) and (21) are calculated on the basis of that projection $S_{i\,max}'$ of the surface element onto the coordinate planes which has the largest area.

In practice this is realized in the program by the appropriate change in the naming of the coordinates so that the projection $S_{i\,max}'$ lies in the zy plane each time.

The signs of the integrals calculated from Eqs. (20) and (21) depend on the direction of numbering, and therefore the modulus of the calculated value of the integrals should be taken in the calculation.

The method described for the solution of the three-dimensional problem of heat conduction was realized in the form of a program for an M-222 computer. With a number of areas $n = 250$ of the partition of the blade surface the time for the formation and solution of the system of equations is 3 h 40 min while the time of calculation of the temperature at one internal point is 45 sec.

The method developed was tested by comparing the results of the calculation with experimental data obtained by blowing through blades with combined cooling (convective + film cooling) at the hot wall. A description of the experimental subject is given in [7].

The measured and calculated temperatures were compared for the following operating parameters: $T_g^* = 905.7°K$, $P_g^* = 1.37 \cdot 10^5$ N/m²; $T_{a.in}^* = 373°K$; $\bar{G}_a = G/G_g = 3\%$.

The results of temperature measurements at the surface of the test blade are shown by points in Fig. 3. Curve 1 is a calculation by the two-dimensional method of [2] with the perforations combined in one cross section; 2) by the same method in a cross section where perforations are absent; 3) calculation of the three-dimensional temperature field in a characteristic element of the blade with allowance for the spatial distribution of the perforations.

It is seen from the graphs that in the middle part of the profile, where the character of the temperature field is close to two-dimensional, the results of the calculations by the two-dimensional and three-dimensional theories differ little. In those places where the temperature field has a clearly expressed three-dimensional character (at the edges due to the presence of perforations) the calculations by the two-dimensional theory can lead to considerable errors (up to 20% at the inlet edge in the example under consideration).

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HEAT TRANSFER OF A VERTICAL CYLINDER BY FREE CONVECTION AND RADIATION

Yu. A. Sokovishin and M. V. Shapiro

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The effect of radiation on free convective heat liberation from the surface of a vertical cylinder located in a transparent medium is studied. It is shown that the radiative component of thermal flux equalizes the surface temperature.

Calculation of the thermal regimes of radio electronic devices requires study of heat transfer from high temperature elements to the surrounding medium. In calculating heat liberation from the surfaces of bodies of semiconductor devices, thermoresistors, microconductors, etc., it is necessary to consider the effect of not only transverse curvature on heat transfer, but also the interaction of various forms of heat transfer. Of special interest in electronics is heat transfer to an immobile medium by free convection and radiation. Existing studies of this problem have considered the case of a plane surface and have mainly been performed by approximate methods [1-4].

We will consider free motion of a viscous incompressible gas with constant physical properties in a boundary layer near a vertical cylinder. The gas is considered optically transparent and we neglect the processes of radiation emission, absorption, and scattering. The

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